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Semi-probabilistic assessment of concrete bridge exploiting additional data from experiments and numerical analysis

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ABSTRACT: The paper presents an application of the novel methodology for the assessment of structures using a semi-probabilistic approach exploiting advanced probabilistic modeling and experimental results. The selected existing bridge is represented by a costly finite element model, which reflects the non-linearity of concrete and the construction process. Due to a significant computational burden of each simulation, it is not feasible to perform a Monte Carlo simulation and a semi-probabilistic approach was thus adopted. In this study, we investigate the possibility of a Gram-Charlier expansion described by the first four central moments efficiently obtained directly from Polynomial Chaos Expansion metamodel together with the uncertainty quantification of input random variables described by a joint probability distribution obtained from experimental data combined with prior assumptions from codes. Obtained results are compared to the standard approach assuming a Lognormal probability distribution of structural resistance.

1 INTRODUCTION

Mathematical models of real structures, e.g. bridges, are typically analyzed by computationally expensive non-linear finite element method (NLFEM) reflecting material and geometrical non-linearity. Non-linear models are not compatible with standard partial safety factors (PSF) implemented in Eurocode (CEN 2002) and advanced probabilistic methods should be employed. Nonetheless, standard probabilistic design or assessment of structures represented by computational models solved by NLFEM is extremely time-consuming and it is usually necessary to use semi-probabilistic methods developed for NLFEM. The paper focuses on the semi-probabilistic assessment of concrete structures using simplified methods.

In the semi-probabilistic approach (Val et al. 1997, Novák & Novák 2021), the resistance of structure R is separated (similarly is in PSF by sensitivity factor α), and the design value R_d that satisfies safety requirements is evaluated, instead of the direct calculation of failure probability. The whole process represents the estimation of a quantile satisfying the given safety requirements under the prescribed simplifying assumptions. The given task is thus simplified to statistical analysis of target probability distribution of resistance (output of the model) – its mean value μ, coefficient of variation (CoV) etc. Safety requirements are given by codes in form of the target reliability index β dependent on consequence classes, e.g. β for the ultimate limit state, moderate consequences of failure and a reference period of 50 years is set at β = 3.8 according to the Eurocode 1990 (CEN 2002). In this paper, we investigate the role of simplifying assumptions regarding the probability distribution of input variables and resistance (output variable).

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The procedure is a combination of the following steps:

- development of NLFEM finite element model of structure (high-fidelity model, computationally very expensive);
- stochastic model based on prior knowledge for input random variables;
- Bayesian approach - based on experimental data updating statistics of input random variables;
- development of surrogate model using Polynomial Chaos Expansion (PCE – low fidelity model, computationally cheap);
- determination of design value of resistance based on statistical moments of resistance directly obtained form PCE or estimated by Monte Carlo using surrogate model using Gram-Charlier expansion.

2 ASSUMPTIONS IN SEMI-PROBABILISTIC APPROACH

Existing simplified semi-probabilistic methods were developed for an estimation of CoV using very low number of samples (ECoV methods), e.g. ECoV by Červenka (Červenka 2013), Taylor Series Expansion (Novák & Novák 2020) or recently developed Eigen ECoV (Novák & Novák 2021). These methods are based on very strict assumptions, which allow to use simple formulas together with a few numerical simulations (e.g. 2 for ECoV by Červenka or 3 for Eigen ECoV) for an estimation of the first two statistical moments. The mean value μ and variance σ^2 are further used to describe an assumed 2-parametric probability distribution of resistance, typically Lognormal distribution or Gaussian distribution. Lognormal distribution is moreover recommended as a typical distribution for modeling of resistance in codes (fib 2013, CEN 2002, JCSS 2001). This has well-justified rationale: Lognormal distribution is non-negative (reflecting reality) and it is fully-described by the first two statistical moments (computational efficiency). However, this paper presents methodology for semi-probabilistic approach for medium-size experimental design (ED) 10-100 samples. In that case, it is possible to construct a surrogate model sufficiently accurate for an estimation of higher statistical moments. Additionally it will be shown that it is beneficial to use Bayesian updating of input variables to estimate a realistic R_d incorporating real data obtained from material experiments for input random variables.

2.1 *Standard approach*

The standard formula for the estimation of R_d, assuming a Lognormal distribution of R, is

$$R_d = \mu_R \cdot exp(-\alpha_R \beta v_R), \tag{1}$$

where μ_R is the mean value, v_R is the coefficient of variation (CoV) and α_R represents sensitivity factor derived from First Order Reliability Method (FORM); the recommended value is α_R = 0.8 according to Eurocode 1990 (CEN 2002). In this case, it is necessary to estimate only the first two statistical moments μ and σ^2. Estimation of statistical moments using ECoV methods is based on numerical simulations with specific quantile of input random variables, e.g. mean values and characteristic values of material parameters. Although such an approach is extremely efficient, it is also very limited to assumed Lognormal distribution of resistance. There are many studies investigating this approach and comparing various ECoV methods (Schlune et al. 2011, Bagge 2020, Novák et al. 2022). Although ECoV methods are well-suited for extremely computational expensive numerical models, their limitations could lead to inaccurate results as probability distribution of resistance can differ from Lognormal distribution significantly in some cases (eg. high non-linearity). Thus the further paragraphs describe a methodology based on Polynomial Chaos and Gram-Charlier Expansions used for estimation of higher statistical moments and construction of an artificial probability distribution for structural resistance.

2.2 *Polynomial chaos and gram-charlier expansions*

An approximation of cumulative distribution function (CDF) F_R of structural resistance R by Gram-Charlier expansion (G-C) is a completely determined by the first four statistical moments obtained here efficiently from Polynomial Chaos Expansion (PCE). Asssuming that it is possible to write probability distribution of R as a perturbation of Gaussian Gaussian probability distribution function (PDF) ϕ. Once the R is normalized to be zero-mean and unit-variance, it is possible to write the Gram-Charlier approximation of CDF in the terms of its higher central moments (skewness γ_Y and kurtosis κ_Y) as:

$$F_R = \Phi(r) - \left[\frac{\gamma_Y}{3\sqrt{2!}} H_2(r) + \frac{\kappa_Y - 3}{4\sqrt{3!}} H_3(r)\right] \phi(r). \tag{2}$$

where $H_n(r)$ are probabilists' Hermite polynomials of n-th order and $\Phi(r)$ represents standard Gaussian CDF.

It is typically not feasible to get higher statistical moments by crude Monte Carlo simulation due to its computational demands, moreover the moments estimated from samples are highly sensitive to outliers. Fortunately, it is possible to get statistical moments analytically in case of PCE, which represents the output variable R as a function g^{PCE} of an another random variable ξ called the germ with given distribution and representing the original computational model R = $g(X)$ via polynomial expansion. A set of polynomials, orthonormal with respect to the probability distribution of the germ, are used as a basis of the Hilbert space of all real-valued random variables of finite variance. In the case of $\boldsymbol{X}$ and ξ being vectors containing M random variables, the polynomial $\Psi(\xi)$ is multivariate and it is built up as a tensor product of univariate orthogonal polynomials:

$$R = g(X) = \sum_{\alpha \in \mathbb{N}^M} \beta_\alpha \Psi_\alpha(\xi), \tag{3}$$

where $\alpha \in \mathbb{N}^M$ is a set of integers called the multi-index corresponding to polynomial orders in each term of PCE, β_α are deterministic coefficients and Ψ_α are multivariate orthogonal polynomials. Coefficients β_α can be usually obtained by ordinary least squares.

Once a PCE approximation is created, it is possible to obtain statistical moments of R analytically, which represents an enormous advantage with respect to this study, as will be shown in computationally expensive numerical example. Specifically, the first statistical moment (mean value) is equal to the first deterministic coefficient of the expansion

$$\mu_R = \langle Y^1 \rangle = \beta_0. \tag{4}$$

Further the variance $\sigma_Y^2 = \langle Y^2 \rangle - \mu_Y^2$ is obtained as a sum of all squared deterministic coefficients except the intercept, which represents the mean value:

$$\sigma_R^2 = \sum_{\substack{\alpha \in A \\ \alpha \neq 0}} \beta_\alpha^2. \tag{5}$$

Higher statistical central moments, skewness γ_R (3^{rd} moment) and kurtosis κ_R (4^{th} moment), can be similarly obtained analytically for Legendre and Hermite polynomials (Novák 2022).

2.3 *Bayesian approach*

Given some experimental data $\mathcal{D}$ for input model parameters, a parameterized model for the data (likelihood function) $p(\mathcal{D}|\theta)$, and a prior probability density $p(\theta)$ for the model parameters, the posterior probability density function (PDF) $p(\theta|\mathcal{D})$ of the model can be identified by Bayesian theorem:

$$p(\theta|\mathcal{D}) = \frac{p(\mathcal{D}|\theta)p(\theta)}{p(\mathcal{D})}. \tag{6}$$

Although Bayes' rule looks simple, its efficient evaluation is still challenging and it must be calculated numerically, and thus Markov Chain Monte Carlo sampling (MCMC) is employed in this paper. For numerical calculation, we use existing algorithm implemented in UQPy package for Python (Olivier et al. 2020). Such an approach allows updating prior knowledge about the material characteristics (Rózsás et al., 2022). In this study, Bayesian approach is employed for updating of materials' statistics (mean and variance) obtained from codes combining prior knowledge and results obtained from laboratory experiments. Obtained updated statistics of input random variables together with prescribed probability distribution function were further used in Monte Carlo simulation using surrogate model in form of PCE. Note that evaluation of PCE is very fast even for very large number of simulations used for estimation of higher statistical moments. Estimated statistical moments were ultimately used for G-C expansion and an estimation of R_d as described in section 2.2.

3 NUMERICAL APPLICATION: POST-TENSIONED CONCRETE BRIDGE

The proposed methodology is applied for the existing post-tensioned concrete bridge with three spans. The super-structure of the mid-span anayzed by NLFEM is 19.98 m long with total width 16.60 m. In transverse direction, each span is constructed from 16 prefabricated bridge girders KA-61 commonly used in Czech Republic. Load is applied according to national annex of Eurocode for load-bearing capacity of road bridges by exclusive loading (by six-axial truck).

3.1 *Finite element model*

The numerical model is created in software ATENA Science based on theory of non-linear fracture mechanics (Červenka & Papanikolaou 2008). In order to reflect complex behavior of the bridge, the numerical model contains three construction phases as illustrated in Figure 1. The NLFEM consists of 13,000 elements of hexahedra type in the major part of the volume and triangular 'PRISM' elements in the part with complicated geometry. Reinforcement and prestressing tendons are represented by discrete 1D elements with geometry according to original documentation. The numerical model is further analysed in order to investigate the ultimate limit state (ULS) (peak of a load-deflection diagram) in order to determine the load-bearing capacity of the bridge. Load-deflection diagram from simulation using mean values of input random variables can be seen in Figure 2 together with typical crack pattern and highlighted 3 limit states: decompression, the first occurrence of cracks and ULS represented by collapse of the bridge.

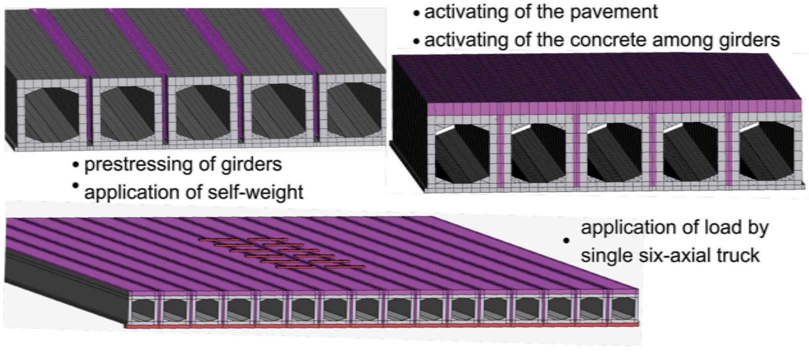

Figure 1. Three construction phases of the bridge mid-span analysed by NLFEM.

3.2 *Stochastic model*

The stochastic model contains 4 random material parameters of concrete C50/60: Young's modulus E; compressive strength of concrete f_c; tensile strength of concrete f_{ct} and fracture energy G_f. Characteristic values of E, f_{ct}, G_f were determined from f_c according to formulas implemented in the fib Model Code 2010 (fib 2013) – G_f, E, and prEN 1992-1-1: 2021 (CEN 2021) – f_{ct}. The last random variable P represents prestressing losses according to JCSS: Probabilistic Model Code (JCSS 2001). The stochastic model is summarized in Table 1. Mean values and coefficients the of variation were obtained according to prEN 1992-1-1: 2021 (Annex A) for adjustment of partial factors for materials. Statistical correlation among randome variables was not considered in this study.

Table 1. Stochastic model of the numerical example.

Var.	Mean	CoV [%]	Distrib.	Units
f_c	56	16	Lognormal	[MPa]
f_{ct}	3.64	22	Lognormal	[MPa]
E	36	16	Lognormal	[GPa]
G_f	195	22	Lognormal	[Jm2]
P	20	30	Normal	[%]

3.3 *Results*

Once the stochastic model was defined and computational model was developed in ATENA Science, it was possible to create 30 realizations of input random vector generated by Latin Hypercube Sampling (Iman & Conover 1980, Novák et al. 2014, 2022), which covers the whole design domain, and thus it is suitable technique for construction of ED for surrogate modeling. Note that each simulation takes approximately 24 hours using standard hardware. The PCE is created with maximum polynomial order $p = 5$. The whole algorithm of adaptive construction of PCE connecting state of art techniques into stand-alone software tool can be found in (Novák & Novák 2018). The design values of resistance R_d are determined as a quantile of distribution of R with identified statistical moments and target reliability indices β_{ULS} = 3.8 according to EN 1990. Additionally, design values are reduced by global safety factor reflecting model uncertainties γ_{R_d} = 1.06 introduced originally in fib Model Code 2010. Note that we compare three design values obtained by described semi-probabilistic approach: i) standard approach assuming Lognormal distribution of R parameterized by the first two statistical moments obtained by PCE; ii) R_d as a quantile of artificial probability distribution constructed by G-C parametrized by the first four statistical moment obtained by PCE and iii) combination of G-C expansion of R and Bayesian updating of input parameters.

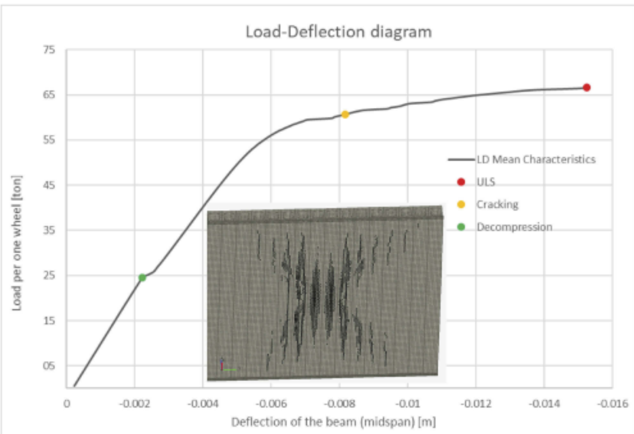

Figure 2. Design values of resistance obtained by semi-probabilistic approach determined by the described methods together with corresponding PDFs.

Bayesian updating is performed using artificially generated data: 20 experiments of concrete specimens. Note that artificially generated data have realistic CoVs of material parameters, which were identified in the previous experimental campaigns (Slowik et al. 2021). Prior distribution of material characteristic was assumed to be Uniform and likelihood distribution is selected according to Table 1, i.e. distributions provided in codes. Obtained results can be found in Figure 3: each row corresponds to a specific material characteristic, the first column shows estimation of mean value and the second column shows estimation of standard deviation. Both columns, the first and second, show also prior and posterior distribution identified by Bayesian approach. Vertical solid lines corresponds to values assumed by codes, obtained directly from experiments by statistical processing and the Bayesian estimation identified as a mean value of posterior distribution. The very last column shows 5000 samples used in MCMC for estimation of posterior distributions. Note that the identified values are significantly different in comparison to recommended values by codes, but this is dependent on real-life experimental results.

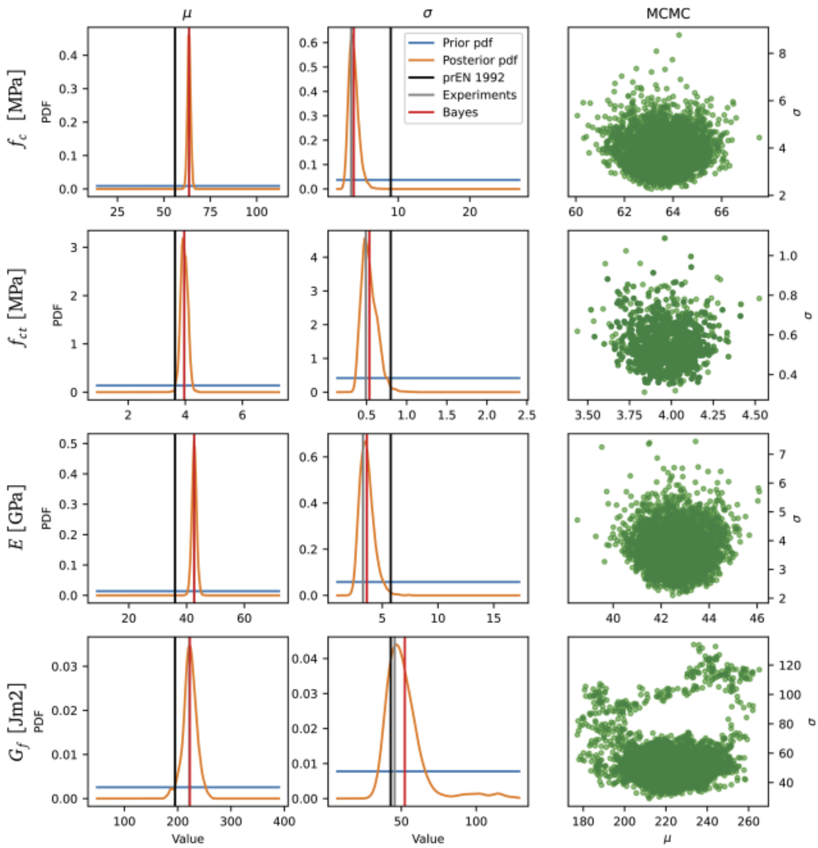

Figure 3. Bayesian estimation of mean μ and standard deviation σ of concrete material characteristics. Solid vertical lines show values determined by codes, experiments and Bayesian approach.

Once the PCE was created, it was possible to analytically derive statistical moments used for Lognormal (first two statistical moments) and G-C expansion (first four statistical moments). The difference between these two design values (corresponding to the identical percentile) is caused by higher statistical moments. Further, the created PCE was employed as computationally cheap surrogate model for crude Monte Carlo simulation with 106 realizations of input random variables identified by Bayesian approach. So Bayes G-C expansion was based on estimation of first four statistical moments needed for Gram-Charlier expansion by Monte Carlo simulation. Comparison of identified PDFs together with determined design values of resistance R_d can be found in 4.

4 CONCLUSIONS

The paper described the advanced semi-probabilistic methodology based on G-C and PCE for estimation of higher statistical moments and an approximation of probability distribution of structural resistance. Additionally, estimation was further improved by Bayesian estimation of input random variables combining likelihood distributions from codes with material experiments. The whole methodology was applied for an estimation of design value of resistance of existing post-tensioned concrete bridge. It can be seen from comparison of determined design values, that is beneficial to include additional information on structural parameters (Bayesian approach) as well as higher statistical moments (G-C expansion). Methodology was shown using concrete bridge example, exploiting additional experimental data and Bayesian approach resulted in less conservative design value here.

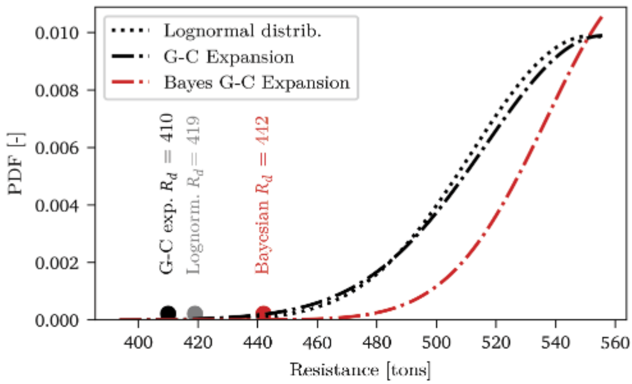

Figure 4. Design values of resistance obtained by semi-probabilistic approach determined by the described methods together with corresponding PDFs.

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